Answers

13.1 $\chi^2 = \sum \frac{(o-e)^2}{e}$

$$= \frac{(352-400)^2}{400} + \frac{(418-400)^2}{400} + \frac{(434-400)^2}{400} + \frac{(480-400)^2}{400} + \frac{(341-400)^2}{400} + \frac{(375-400)^2}{400}$$

=5.76 + 0.81 + 2.89 + 16 + 8.7025 + 1.5625 = 35.725.
The decision rule is:
If $\chi^2 > 11.0705$, reject Ho;
Otherwise, do not reject Ho.
Because $\chi^2 = 35.725 > 11.0705$, reject the null hypothesis.
We conclude, based on the sample information and the results of the goodness-of-fit test that the die is not fair. The distributions of outcomes for this die are not uniformly distributed.

13.2.

x	o Frequency	Poisson Probability	e Expected Frequency
2 or less	7	0.0620	30.98
3	29	0.0892	44.62
4	26	0.1339	66.93
5	52	0.1606	80.31
6	77	0.1606	80.31
7	77	0.1377	68.84
8	72	0.1033	51.63
9	53	0.0688	34.42
10	35	0.0413	20.65
11	28	0.0225	11.26
12	18	0.0113	5.63
13	13	0.0052	2.60
14 or more	13	0.0036	1.81
Total	500	1.0000	500

Now you need to check to see if any of the expected cell frequencies are less than 5. In this case we see that there are two instances where this is the case. To deal with this, you should collapse categories so that all expected frequencies are at least 5. Doing this gives the following:

x	o Frequency	Poisson Probability	e Expected Frequency
2 or less	7	0.062	29.39
3	29	0.0892	42.28
4	26	0.1339	63.47
5	52	0.1606	76.12
6	77	0.1606	76.12
7	77	0.1377	65.27
8	72	0.1033	48.96
9	53	0.0688	32.61
10	35	0.0413	19.58
11	18	0.0225	10.67
12 or more	44	0.0201	9.53
Total	474	1	474

Now we can compute the chi-square test statistic using equation 13-1 as follows”

$$\chi^2 = \sum \frac{(o-e)^2}{e} = \frac{(7-29.39)^2}{29.39} + \frac{(29-42.28)^2}{42.28} + \ldots + \frac{(9.53-44)^2}{9.53} = 218.62$$

Because $\chi^2 = 218.62 > 18.0370$, we reject the null hypothesis.

The population distribution is not Poisson distributed with a mean of 6.

13.3

# of Defective Batteries Per Package	*Observed (o)*	*Binomial Probability n=50, p = 0.02*	*Expected Frequency (e)*	*$(o_i-e_i)^2/e_i$*
0	165	0.36417	145.668	2.5656
1	133	0.37160	148.641	1.6458
2	65	0.18580	74.320	1.1688
3	28	0.06067	24.268	0.5740
4 or more	9	0.01776	7.103	0.5065
Total	400			6.4607

The calculated chi-square test statistic is $\chi^2 = 6.4607$.

The decision rule is:
If $\chi^2 > 13.2767$, reject Ho;
Otherwise, do not reject Ho.
Because $\chi^2 = 6.4607 < 13.2767$, do not reject the null hypothesis.

We conclude, based on the sample information and the results of the goodness-of-fit test that the binomial distribution with n = 50 and $\pi = 0.02$ is an appropriate distribution for describing the company's sampling plan.

13.16

a. H_O: The row and column variables are independent
H_A: The row and column variables are not independent

b. The following contingency table shows the results of the sampling

	C_1	C_2	Total
R_1	51	207	258
R_2	146	185	331
R_3	240	157	397
Total	437	549	986

The expected cell frequencies are determined by

$$e_{ij} = \frac{\text{(row total)(column total)}}{\text{grand total}}$$. As an example $e_{11} = \frac{437(258)}{986} = 114.359$.

The expected cell values for all cells are

	C_1	C_2	Total
R_1	114.35	143.65	258
R_2	146.70	184.30	331
R_3	175.95	221.05	397
Total	437	549	986

c. The test statistic is computed using Equation 13.2

$$\chi^2 = \sum\sum \frac{(o_{ij}-e_{ij})^2}{e_{ij}} = \frac{(51-114.35)^2}{114.35} + \ . \ . \ . + \frac{(157-221.05)^2}{221.05} = 104.905$$

d. The critical value for this test will be the chi-square value with (r-1)(c-1) = (3 -1)(2-1) = 2 degrees of freedom with $\alpha = 0.05$. From Appendix G, the critical value is 5.9915. Because $\chi^2 = 104.905 > 5.9915$, reject the null hypothesis. The row and column variables are not independent.

e. Since $\chi^2_{0.005}$ = 10.5965 < χ^2 = 104.905, then p-value < 0.005. The exact p-value can be found using Excel's CHIDIST or Minitab's CALC>PROBABILITY DISTRIBUTIONS command to be essentially 0.

13.21.

H_0 : The grade a student receives in the class is independent of the seat location in the class.

H_A : The grade received is not independent of seat location

The contingency table with expected frequencies included is:

	A	B	C	D	F	Total
Front	18	55	30	3	0	106
	7.42	**29.68**	**60.685**	**7.42**	**0.795**	
Middle	7	42	95	11	1	156
	10.92	**43.68**	**89.31**	**10.92**	**1.17**	
Back	3	15	104	14	2	138
	9.66	**38.64**	**79.005**	**9.66**	**1.035**	
Total	28	112	229	28	3	400

We have some expected cell frequencies that are smaller than 5. Before collapsing categories, we will see if the null hypothesis is rejected. If not, then we need not worry about the small expected frequencies. Then the test statistic is:

$$\chi^2 = \sum\sum\frac{(o-e)^2}{e} = 87.3$$

Because $\chi^2 = \sum\sum\frac{(o-e)^2}{e} = 87.3 > 15.507$ we would reject the null hypothesis. Because we reject, we need to take care of the small expected frequencies. We will do this by combining the D and F grades with the revised contingency table as follows:

	A	B	C	D&F	Total
Front	18	55	30	3	106
	7.42	29.68	60.685	8.22	
Middle	7	42	95	12	156
	10.92	43.68	89.31	12.09	
Back	3	15	104	16	138
	9.66	38.64	79.005	10.7	
Total	28	112	229	31	

The revised test statistic is:

$$\chi^2 = \sum\sum\frac{(o-e)^2}{e} = 86.9$$

and the revised critical value now has (3-1)(4-1) = 6 degrees of freedom and is 12.5916. Therefore,

Because $\chi^2 = \sum\sum\frac{(o-e)^2}{e} = 86.9$ >12.5916, we reject the null hypothesis.

The instructor should conclude that course grade is related to seating location.

13.22

H_0: Stock price changes today are independent of previous day price changes.

H_A: Stock price changes today are not independent of previous day price changes.

$\alpha = 0.05$

		Observed Frequencies			
		Price Change Previous Day			
		Up	No Change	Down	Total
Price Change Today	Up	14	16	12	42
	No Change	6	8	6	20
	Down	16	14	8	38
	Total	36	38	26	100

		Expected Frequencies			
		Price Change Previous Day			
		Up	No Change	Down	Total
Price Change Today	Up	15.12	15.96	10.92	42
	No Change	7.2	7.6	5.2	20
	Down	13.68	14.44	9.88	38
	Total	36	38	26	100

$$\chi^2 = \sum\sum\frac{(o-e)^2}{e} = \frac{(14-15.12)^2}{15.12} + \frac{(16-15.96)^2}{15.96} + \cdots + \frac{(8-9.88)^2}{9.88} = 1.2987$$

The chi-square critical value for (r-1)(c-1) = (3-1)(3-1) = 4 degrees of freedom and alpha = 0.05 is 9.4877. Since the calculated chi-square value is less than the critical chi-square value, we do not reject the null hypothesis and conclude that daily stock price changes are independent.