Answers:

1. The starting point in the random numbers table for column 14, row 10 is the value 0. We need two digit numbers since we have sixty nurses in the population. Thus, the first two digit random number is 02 followed by (going down the table) 01, 38, etc. Keep in mind that if a random number exceeds 60, we skip it and go to the next number. The following is the sequence of random numbers that we will use:

02, 01, 38, 59, 34, 50

Now the overtime hours reported by these six nurses are:

1 2 0 5 4 6

The sample mean for this sample is:

$$\bar{x} = \frac{\sum x}{n} = \frac{18}{6} = 3$$

To determine the sampling error, we first compute the population mean as:

$$\text{«} = \frac{\sum x}{n} = \frac{206}{60} = 3.43$$

The sampling error is:

Sampling error = $\bar{x} - \text{«} = 3 = 3.43 = -.43$ hours

If the administrator plans to analyze the justification for the overtime reported by these six nurses, he can feel comfortable knowing that the sample mean is quite close to the population mean.

2.

a. P($\bar{x}$ < 400,000) = P(z < (400,000 – 417,330)/[(45,285/ $\sqrt{40}$] $\sqrt{\frac{220-40}{219}}$ = P(z < -2.67) = 0.5 – 0.4962 = 0.0038

b. The shape of the sampling distribution of the sample means will be approximately normally distributed for samples of size 40 regardless of the shape of the population from which the samples are taken. This characteristic of the distribution of the sample means is known as the Central Limit Theorem.

c. The standard deviation of the distribution of the sample mean = 45,285/ $\sqrt{40}\sqrt{\frac{220-40}{219}}$ = 6,491.40

Note, the standard deviation is modified using the finite correction factor since the sample size is more than 5% of the size of the population.

d. P($\bar{x}$ < 400,000) = P(z < (400,000 – 417,330)/(45285/ $\sqrt{60}\sqrt{\frac{220-60}{219}}$) = P(z < -3.49) = 0.5 – 0.5 = approximately 0.00. The standard deviation of the distribution of the sample mean = 45,285/ $\sqrt{60}\sqrt{\frac{220-60}{219}}$ = 4,997.08 . The distribution does not change shape. It will still be approximately normal.

e. Having a probability of 0.15 in the upper tail results in a z of 1.04
1.04 = (x – 417,330)/45285; x = $464,426.40 is the smallest sales level for the sales personnel to enjoy a complementary trip to Hawaii

3.

The following steps can be used to answer this question:

Step 1: Determine the sample mean.

$$\bar{x} = \frac{\sum x}{n} = \frac{469}{14} = 33.5$$

Step 2: Define the sampling distribution.

The sampling distribution will be normally distributed and will have $\text{«}_{\bar{x}} = \text{«} = 34.3$ and a standard deviation equal to $\pm_{x} = \frac{\pm}{\sqrt{n}} = \frac{5.7}{\sqrt{14}} = 1.52$

Step 3: Define the event of interest.

We are interested in the following:

$$P(\bar{x} \leq 33.5) = ?$$

Step 4: Convert the sample mean to a standardized z value.

$$z = \frac{\bar{x} - \text{«}}{\frac{\pm}{\sqrt{n}}} = \frac{33.5 - 34.3}{\frac{5.7}{\sqrt{14}}} = \frac{-0.8}{1.52} = -0.53$$

Step 5: Use the standard normal distribution to find the desired probability.

The probability associated with a z-value of -0.53 from the standard normal table is 0.2019 Then

$$P(\bar{x} \leq 33.5) = 0.5000 - 0.2019 = 0.2981$$

Thus, there is slightly less than a 30% chance that a random sample of size 14 could produce a sample mean equal to 33.5 or less if the sample came from a population with mean equal to 34.3 and standard deviation equal to 5.7. The organization would probably agree the average weight of bags has not changed.

4.

a. The population is specified to have $\pi = 0.35$. This indicates that the standard error of sample proportion is equal to $\sqrt{\frac{\text{®}(1-\text{®})}{n}} = \sqrt{\frac{0.35(1-0.35)}{100}} = 0.048$. The distance between the sample and population proportion is $p - \pi$. We have z = $\frac{p - \text{®}}{\sqrt{\frac{\text{®}(1-\text{®})}{n}}}$. Solving for $p - \pi$, we obtain $p - \pi = z\sqrt{\frac{\text{®}(1-\text{®})}{n}}$. Therefore, $p - \pi = \pm 0.05 = z\sqrt{\frac{\text{®}(1-\text{®})}{n}} = z(0.048)$. Then $z = \pm\frac{0.05}{0.048}$; $z = \pm 1.04$.

Using the standard normal distribution table to determine the desired probability, we obtain

$P(-1.04 \leq z \leq 1.04) = P(-1.04 \leq z \leq 0) + P(0 \leq z \leq 1.04) = 2(0.3508) = 0.7016$

b. The standard error of sample proportion is equal to $\sqrt{\frac{\text{®}(1-\text{®})}{n}} = \sqrt{\frac{0.35(1-0.35)}{100}} = 0.048$. The difference $p - \pi$ equaling one standard error would indicate $p - \pi = 0.048 = z\sqrt{\frac{\text{®}(1-\text{®})}{n}} = z(0.048)$.

Then $z = \pm 1.00$. Using the standard normal distribution table to determine the desired probability, we obtain

$P(-1.00 \leq z \leq 1.00) = P(-1.00 \leq z \leq 0) + P(0 \leq z \leq 1.00) = 2(0.3413) = 0.6826$.

c. As in part a., $p - \pi = \pm 0.10 = z\sqrt{\frac{\text{®}(1-\text{®})}{n}} = z(0.048)$. Then $z = \pm\frac{0.10}{0.048}$;

$z = \pm 2.08$. Using the standard normal distribution table to determine the desired probability, we obtain

$P(-2.08 \leq z \leq 2.08) = P(-2.08 \leq z \leq 0) + P(0 \leq z \leq 2.08) = 2(0.4812) = 0.9624$.

5.

The sample proportion is computed using:

$$p = \frac{x}{n}$$

where x is the number of “YES” responses in the sample and n is the sample size. We get:

$$p = \frac{x}{n} = \frac{27}{60} = 0.45$$

b. To find the probability of a sample proportion as extreme or more extreme than that found in part a. we can use the following steps:

Step 1: Determine the population proportion.

The population proportion is $\pi = 0.40$

Step 2: Calculate the sample proportion.

The sample proportion is calculated using $p = \frac{x}{n}$. This was found in part a. to be

$$p = \frac{x}{n} = \frac{27}{60} = 0.45$$

Step 3: Determine the mean and standard deviation of the sampling distribution.   The mean is

$\mu_p = \pi = 0.40$ and the standard deviation of the sampling distribution is

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = \sqrt{\frac{0.40(1-0.40)}{60}} = 0.063$$

Step 4: Define the event of interest.

We are interested in finding:

$$P(p \geq 0.45) = ?$$

Step 5: Convert the sample proportion to a standardized z-value.

$$z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi}{n}}} = \frac{0.45-0.40}{\sqrt{\frac{0.40(1-0.40)}{60}}} = \frac{0.05}{0.063} = 0.79$$

Step 6: Use the standard normal distribution table to determine the probability for the event of interest.

The probability associated with z = 0.79 is 0.2852. This, the desired probability is:

$$P(p \geq .45) = 0.5000 - 0.2852 = 0.2148$$

6.

a. $\pi = 0.105$ $p = 8/50 = 0.16$

$$P(p \geq 0.16) = P(z \geq \frac{0.16-0.105}{\sqrt{\frac{0.105(1-0.105)}{50}}})=P(z\geq 1.27)=0.5–0.3980=0.1020$$

b. $\pi = 0.105$, $p = 5/50 = 0.10$

$$P(p \leq 0.10) = P(z \leq \frac{0.10-0.105}{\sqrt{\frac{0.105(1-0.105)}{50}}})=P(z\leq -0.12)=0.5-0.0478=0.4522$$

c. $\pi = 0.105$, $p = 5/200 = 0.025$

$$P(p \leq 0.025) = P(z \leq \frac{0.025-0.105}{\sqrt{\frac{0.105(1-0.105)}{200}}})=P(z\leq -3.69)=0.5-0.5\approx 0.0$$

7. Because the population standard deviation is unknown, if we can assume that the population distribution is approximately normal, the t-distribution can be used to obtain the critical value for the confidence interval. The following format is used:

Using Excel, the t-value for 90% confidence and n – 1 = 90 degrees of freedom is t = 1.662.
Then the confidence interval estimate is:

$$18.2 \pm 1.662\frac{6.3}{\sqrt{91}}$$

$$18.2 \pm 1.10$$

17.1 --- 19.3

Thus, based on these sample data, with 90 percent confidence we conclude that the mean highway mpg for SUV's is in the range 17.1 mpg to 19.3 mpg.

8.

a. $\bar{x} \pm z\frac{\pm}{\sqrt{n}} = 5000 \pm 1.96(\frac{1500}{\sqrt{179}})$ = 5000 ± 219.75 = (4780.25, 5219.75)

b. The margin of error is $z\frac{\pm}{\sqrt{n}} = 1.96(\frac{1500}{\sqrt{179}})$ = 219.75.

c. Reducing the margin of error by 50 percent would produce a margin of error = 219.75/2 = 109.875. So $z\frac{\pm}{\sqrt{n}} = 1.96(\frac{1500}{\sqrt{n}})$ = 109.875.

Solving n = $\left(\frac{1.96(1500)}{109.875}\right)^2$ = 715.97 ≈ 716.

9.

a. The best estimate of the true mean is the sample mean. $\bar{x} = \frac{\sum x}{n} = \frac{630}{36} = \$17.50$.

b. Because the population standard deviation is unknown, the z-distribution cannot be used. However, because the sample size is large (≥ 30), the t-distribution is used. Because the population standard deviation is unknown, the standard error of the sampling distribution is estimated using $\frac{s}{\sqrt{n}}$. The sample standard deviation is computed using the formula s = $\sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$ = 7.52. The standard error = $\frac{7.52}{\sqrt{36}}$ = 1.25.

Using Excel, the degrees of freedom = 36-1 = 35. The t-value = 2.03. The 95% confidence level is

$$\bar{x} \pm t\frac{s}{\sqrt{n}} = 17.50 \pm 2.03\frac{7.52}{\sqrt{36}} = 17.50 \pm 2.54 = 14.96\text{-------------}20.04.$$

c. Yes. While the sample mean of \$17.50 is less than the average of \$19.00 stated by the manager, the sample mean is subject to sampling error. One would not expect a specific $\bar{x}$ to equal the true mean. However, the 95% confidence interval has limits of \$14.96 and \$20.04, which contain the \$19.00 stated by the manager. Therefore, the statement made by the concessions manager is consistent with the results found in part (b).

10.

a. 0.38 ± 2.575($\sqrt{[(0.38)(1-0.38]/499}$); 0.324 ----- 0.436

b. Since pilot sample information is not given, a conservative estimate must be made:
$n = 1.96^2(0.5)(1 – 0.5)/(0.01)^2 = 9,604$

11. p = 345/1000 = 0.345

a. 0.345 ± 1.96($\sqrt{[(0.345)(1-0.345]/1000}$); 0.3155 ----- 0.3745

Based on the sample data and with 95% confidence we believe that the proportion of customers that carry more than one bag on the airline is between 0.3155 and 0.3745. The proportion of customers who would have been affected by the policy is thought to be in this range.

b. (0.3155)(568) ----- (0.3745)(568); 179.20 ----- 212.716

c. p = (280/690) = 0.4058

0.4058 ± 1.96($\sqrt{[(0.4058)(1-0.4058]/690}$); 0.3692 ---- 0.4424

Based on 95% confidence, the proportion of males that would be affected is between .3692 and .4424. So less that half would be affected. However, this might imply that men may be more affected than the population overall.

d. $n = \dfrac{z^2 p(1-p)}{e^2} = \dfrac{1.96^2(.15)(1-.15)}{.02^2}$ = 1,224.51 = 1,225

12.

a. $p = \dfrac{x}{n} = \dfrac{35}{300}$ = 0.1167. This is a characteristic of the sample. Even though it is smaller than the 14.1% attained by the population as a whole, this could be just a result of sampling error. An inference based upon a sample must incorporate probability in the analysis. Here, it is required to determine the probability that an event this extreme or more so has occurred. This is calculated in part b.

b. $n\pi = 300(0.141) = 42.3 > 5$; $n(1 - \pi) = 300(1 - 0.141) = 257.7 > 5$. Therefore, the sampling distribution of p can be approximated with a normal distribution.

$$P(p \le 0.1167) = P\left(z \le \frac{p - ®}{\sqrt{\frac{®(1-®)}{n}}}\right) = P\left(z \le \frac{0.1167 - 0.141}{\sqrt{\frac{0.141(1-0.141)}{300}}}\right) = P(z \le -1.21) = 0.5 – 0.3869 =$$

0.1131. This indicates that there is a somewhat significant probability that a result at most as large as our sample could occur from a population with a proportion = 0.141.

c. $p \pm z\sqrt{\dfrac{p(1-p)}{n}} = 0.1167 \pm 2.33\sqrt{\dfrac{0.1167(1-0.1167)}{300}}$ = 0.1167 ±0.0431 = (0.0736, 0.1598). Since 0.141 is enclosed by this confidence interval, it is quite plausible that the proportion of students that pass at least one AP class in math and science is not different from those that pass at least one AP class.