QM 350

Operations Research

Chapter 7

Problem 2:

Suppose that a decision maker faced with four decision alternatives and four states of nature develops the following profit payoff table:

	State of Nature			
Decision Alternative	s_1	s_2	s_3	s_4
d_1	14	9	10	5
d_2	11	10	8	7
d_3	9	10	10	11
d_4	8	10	11	13

a. If the decision maker knows nothing about the probabilities of the four states of nature, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?

b. Which approach do you prefer? Explain. Is establishing the most appropriate approach before analyzing the problem important for the decision maker? Explain.

c. Assume that the payoff table provides cost rather than profit payoffs. What is the recommended decision using the optimistic, conservative, and minimax regret approaches?

Problem 4:

Amy Lloyd is interested in leasing a new Saab and has contacted three automobile dealers for pricing information. Each dealer offered Amy a closed-end 36-month lease with no down payment due at the time of signing. Each lease includes a monthly charge and a mileage allowance. Additional miles receive a surcharge on a per-mile basis. The monthly lease cost, the mileage allowance, and the cost for additional miles follow:

Dealer	Monthly Cost	Mileage Allowance	Cost per Additional Mile
Forno Saab	$299	36,000	$0.15
Midtown Motors	$310	45,000	$0.20
Hopkins Automotive	$325	54,000	$0.15

Amy decided to choose the lease option that will minimize her total 36-month cost. The difficulty is that Amy is not sure how many miles she will drive over the next three years. For purposes of this decision she believes it is reasonable to assume that she will drive 12,000 miles per year, 15,000 miles per year, or 18,000 miles per year. With this assumption Amy estimated her total costs for the three lease options. For

example, she figures that the Forno Saab lease will cost her $10,764 if she drives 12,000 miles per year, $12,114 if she drives 15,000 miles per year, or $13,464 if she drives 18,000 miles per year.

a. What is the decision, and what is the chance event?

b. Construct a payoff table for Amy's problem.

c. If Amy has no idea which of the three mileage assumptions is most appropriate, what is the recommended decision (leasing option) using the optimistic, conservative, and minimax regret approaches?

d. Suppose that the probabilities that Amy drives 12,000, 15,000, and 18,000 miles per year are 0.5, 0.4, and 0.1, respectively. What option should Amy choose using the expected value approach?

e. Develop a risk profile for the decision selected in part (d). What is the most likely cost, and what is its probability?

f. Suppose that after further consideration Amy concludes that the probabilities that she will drive 12,000, 15,000, and 18,000 miles per year are 0.3, 0.4, and 0.3, respectively. What decision should Amy make using the expected value approach?

Problem 7:

Hudson Corporation is considering three options for managing its data processing operation: continuing with its own staff, hiring an outside vendor to do the managing (referred to as outsourcing), or using a combination of its own staff and an outside vendor. The cost of the operation depends on future demand. The annual cost of each option (in thousands of dollars) depends on demand as follows:

	Demand		
Staffing Options	**High**	**Medium**	**Low**
Own staff	650	650	600
Outside vendor	900	600	300
Combination	800	650	500

a. If the demand probabilities are 0.2, 0.5, and 0.3, which decision alternative will minimize the expected cost of the data processing operation? What is the expected annual cost associated with that recommendation?

b. Construct a risk profile for the optimal decision in part (a). What is the probability of the cost exceeding $700,000?

Problem 8:

The following payoff table shows the profit for a decision problem with two states of nature and two decision alternatives:

Decision Alternative	State of Nature s_1	s_2
d_1	10	1
d_2	4	3

a. Use graphical sensitivity analysis to determine the range of probabilities of state of nature s1 for which each of the decision alternatives has the largest expected value.

b. Suppose $P(s_1) = 0.2$ and $P(s_2) = 0.8$. What is the best decision using the expected value approach?

c. Perform sensitivity analysis on the payoffs for decision alternative d_1. Assume the probabilities are as given in part (b) and find the range of payoffs under states of nature s_1 and s_2 that will keep the solution found in part (b) optimal. Is the solution more sensitive to the payoff under state of nature s_1 or s_2?

Problem 9:

Myrtle Air Express decided to offer direct service from Cleveland to Myrtle Beach. Management must decide between a full-price service using the company's new fleet of jet aircraft and a discount service using smaller capacity commuter planes. It is clear that the best choice depends on the market reaction to the service Myrtle Air offers. Management developed estimates of the contribution to profit for each type of service based upon two possible levels of demand for service to Myrtle Beach: strong and weak. The following table shows the estimated quarterly profits (in thousands of dollars):

Service	Demand for Service: Strong	Weak
Full price	$960	−$490
Discount	$670	$320

a. What is the decision to be made, what is the chance event, and what is the consequence for this problem? How many decision alternatives are there? How many outcomes are there for the chance event?

b. If nothing is known about the probabilities of the chance outcomes, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?

c. Suppose that management of Myrtle Air Express believes that the probability of strong demand is 0.7 and the probability of weak demand is 0.3. Use the expected value approach to determine an optimal decision.

d. Suppose that the probability of strong demand is 0.8 and the probability of weak demand is 0.2. What is the optimal decision using the expected value approach?

e. Use graphical sensitivity analysis to determine the range of demand probabilities for which each of the decision alternatives has the largest expected value.

Answers

2. a.

Decision	Maximum Profit	Minimum Profit
d_1	14	5
d_2	11	7
d_3	11	9
d_4	13	8

Optimistic approach: select d_1

Conservative approach: select d_3

Regret or Opportunity Loss Table with the Maximum Regret

	s_1	s_2	s_3	s_4	Maximum Regret
d_1	0	1	1	8	8
d_2	3	0	3	6	6
d_3	5	0	1	2	5
d_4	6	0	0	0	6

Minimax regret approach: select d_3

b. The choice of which approach to use is up to the decision maker. Since different approaches can result in different recommendations, the most appropriate approach should be selected before analyzing the problem.

c.

Decision	Minimum Cost	Maximum Cost
d_1	5	14
d_2	7	11
d_3	9	11
d_4	8	13

Optimistic approach: select d_1

Conservative approach: select d_2 or d_3

Regret or Opportunity Loss Table

	s_1	s_2	s_3	s_4	Maximum Regret
d_1	6	0	2	0	6
d_2	3	1	0	2	3
d_3	1	1	2	6	6
d_4	0	1	3	8	8

Minimax regret approach: select d_2

4. a. The decision is to choose the best lease option; there are three alternatives. The chance event is the number of miles Amy will drive per year. There are three possible outcomes.

b. The payoff table for Amy's problem is shown below. To illustrate how the payoffs were computed, we show how to compute the total cost of the Forno Saab lease assuming Amy drives 15,000 miles per year.

Total Cost = (Total Monthly Charges) + (Total Additional Mileage Cost)
= 36($299) + $0.15(45,000 - 36,000)
= $10,764 + $1350
= $12,114

	Annual Miles Driven		
Dealer	12,000	15,000	18,000
Forno Saab	$10,764	$12,114	$13,464
Midtown Motors	$11,160	$11,160	$12,960
Hopkins Automotive	$11,700	$11,700	$11,700

c.

Decision Alternative	Minimum Cost	Maximum Cost
Forno Saab	$10,764	$13,464
Midtown Motors	$11,160	$12,960
Hopkins Automotive	$11,700	$11,700

Optimistic Approach: Forno Saab ($10,764)

Conservative Approach: Hopkins Automotive ($11,160)

Opportunity Loss or Regret Table

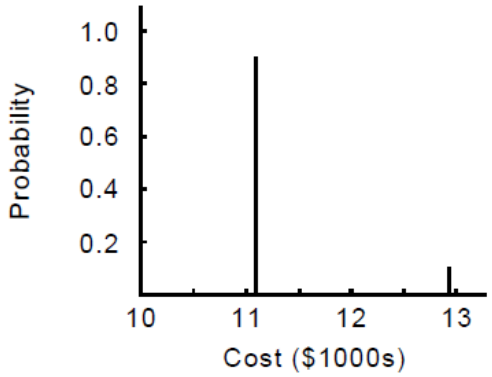
	Actual Miles Driven			
Decision Alternative	36,000	45,000	54,000	Maximum Regret
Forno Saab	0	$954	$1,764	$1764
Midtown Motors	$396	0	$1,260	$1260
Hopkins Automotive	$936	$540	0	$936

Minimax Regret Approach: Hopkins Automotive

d. EV (Forno Saab) = 0.5($10,764) + 0.4($12,114) + 0.1($13,464) = $11,574
EV (Midtown Motors) = 0.5($11,160) + 0.4($11,160) + 0.1($12,960) = $11,340
EV (Hopkins Automotive) = 0.5($11,700) + 0.4($11,700) + 0.1($11,700) = $11,700

Best Decision: Midtown Motors

e.



The most likely cost is $11,160 with a probability of 0.9. There is a probability of 0.1 of incurring a cost of $12,960.

f. EV (Forno Saab) = 0.3($10,764) + 0.4($12,114) + 0.3($13,464) = $12,114
EV (Midtown Motors) = 0.3($11,160) + 0.4($11,160) + 0.3($12,960) = $11,700
EV (Hopkins Automotive) = 0.3($11,700) + 0.4($11,700) + 0.3($11,700) = $11,700

Best Decision: Midtown Motors or Hopkins Automotive

With these probabilities, Amy would be indifferent between the Midtown Motors and Hopkins Automotive leases. However, if the probability of driving 18,000 miles per year goes up any further, the Hopkins Automotive lease will be the best.

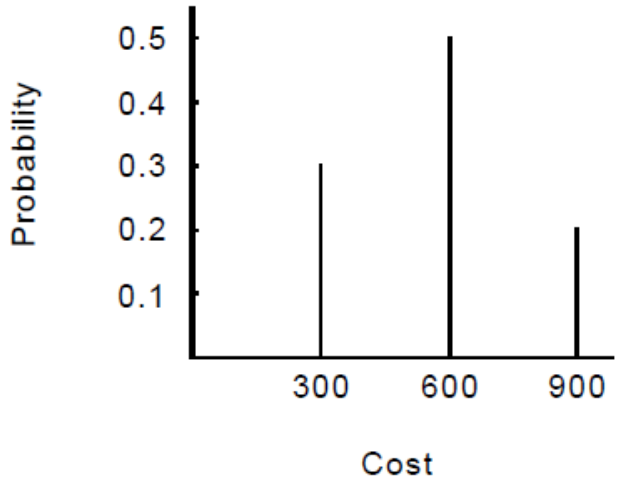
7. a. EV(own staff) = 0.2(650) + 0.5(650) + 0.3(600) = 635
EV(outside vendor) = 0.2(900) + 0.5(600) + 0.3(300) = 570
EV(combination) = 0.2(800) + 0.5(650) + 0.3(500) = 635

The optimal decision is to hire an outside vendor with an expected annual cost of $570,000.

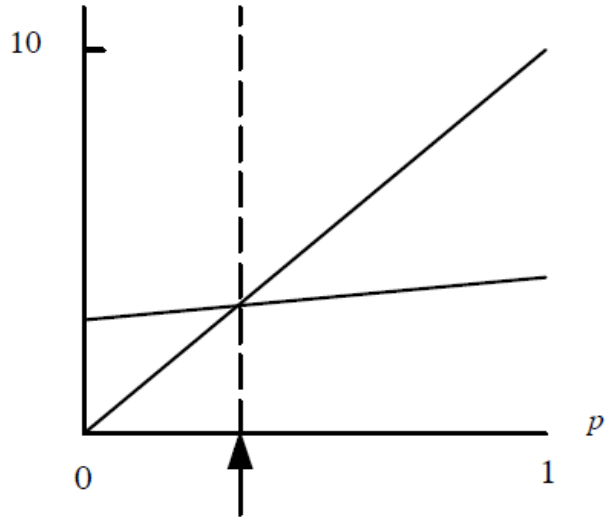
b. The risk profile in tabular form is shown.

Cost	Probability
300	0.3
600	0.5
900	0.2
	1.0

A graphical representation of the risk profile is also shown:



8. a. $EV(d_1) = p(10) + (1 - p)(1) = 9p + 1$
$EV(d_2) = p(4) + (1 - p)(3) = 1p + 3$



Value of p for
which EVs are equal

$9p + 1 = 1p + 3$ and hence $p = .25$

d_2 is optimal for $p \le 0.25$; d_1 is optimal for $p \ge 0.25$.

b. The best decision is d_2 since $p = 0.20 < 0.25$.

$EV(d_1) = 0.2(10) + 0.8(1) = 2.8$
$EV(d_2) = 0.2(4) + 0.8(3) = 3.2$

c. The best decision in part (b) is d_2 with $EV(d_2) = 3.2$. Decision d_2 will remain optimal as long as its expected value is higher than that for d_1 ($EV(d_1) = 2.8$).

Let s = payoff for d_2 under state of nature s_1. Decision d_2 will remain optimal provided that

$EV(d_2) = 0.2(s) + 0.8(3) \ge 2.8$
$0.2s \ge 2.8 - 2.4$
$0.2s \ge 0.4$
$s \ge 2$

As long as the payoff for s_1 is ≥ 2, then d_2 will be optimal.

9. a. The decision to be made is to choose the type of service to provide. The chance event is the level of demand for the Myrtle Air service. The consequence is the amount of quarterly profit. There are two decision alternatives (full price and discount service). There are two outcomes for the chance event (strong demand and weak demand).

b.

Type of Service	Maximum Profit	Minimum Profit
Full Price	$960	-$490
Discount	$670	$320

Optimistic Approach: Full price service
Conservative Approach: Discount service

Opportunity Loss or Regret Table

	High Demand	Low Demand	Maximum Regret
Full Service	0	810	810
Discount Service	290	0	290

Minimax Regret Approach: Discount service

c. EV(Full) = 0.7(960) + 0.3(-490) = 525
EV (Discount) = 0.7(670) + 0.3(320) = 565

Optimal Decision: Discount service

d. EV(Full) = 0.8(960) + 0.2(-490) = 670
EV (Discount) = 0.8(670) + 0.2(320) = 600

Optimal Decision: Full price service

e. Let p = probability of strong demand

EV(Full) = p(960) + (1- p)(-490) = 1450p - 490
EV (Discount) = p(670) + (1- p)(320) = 350p + 320

EV (Full) = EV(Discount)
1450p - 490 = 350p + 320
1100p = 810
p = 810/1100 = 0.7364

If p = 0.7364, the two decision alternatives provide the same expected value.

For values of p below 0.7364, the discount service is the best choice. For values of p greater than 0.7364, the full price service is the best choice.